# Performance Analysis of 3D NDT Scan Matching for Autonomous Vehicles Using INS/GNSS/3D LiDAR-SLAM Integration Scheme

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Abstract—Because robustness and accuracy of localization are crucial for autonomous driving applications. Using the conventional integration scheme of Inertial Navigation System (INS) and Global Navigation Satellite System (GNSS), pose estimation error can drift and accumulate with time, especially in GNSS challenging environment and in unknown environment where an existing map has not been constructed. In this paper, in term of using multi-sensor fusion for improving the positioning accuracy, we proposed a localization method that is based on LiDAR-based 3D Normal Distribution Transform (NDT) scan matching with an INS/GNSS integration scheme. As the experimental results, our proposed method showed a statistical improvement over the state of the art INS/GNSS integration scheme.

Keywords—Autonomous driving, Localization, NDT scan matching, INS/GNSS/NDT

## I. INTRODUCTION

Using conventional INS/GNSS integration scheme sometimes accurate pose estimation cannot be guaranteed because of pose estimation error can drift and accumulate with time in such a challenging environment. To scope with this problem, multi-sensor fusion scheme need to be considered. In particular, using light detection and ranging (LiDAR) that can continuously provide a more stable and accurate ranging information of point clouds. To estimate pose of ego vehicle using LiDAR-based method, featurebased scan matching methods are more popular for LiDARbased simultaneous localization and mapping (SLAM), especially in rich areas where there are many sufficient number of landmarks that can be used to perform scan matching. The popular feature-based algorithms as LiDAR Odometry and mapping (LOAM) [1], and its improved algorithm [2] were proposed and presented. In recent years, the fusion of the GNSS/INS integrated navigation system and feature-based LiDAR-SLAM have been proposed by [3], [4]. However, these methods could only be applied to indoor rather than outdoor environments with low and lack geometric information. In environment where features are limited, using features extracted from the scan of point cloud for scan matching may not work well. To obtain more flexible scan matching method and dealing with problem of those specific geometric information. In this paper, we proposed to use the distribution or mathematical-based scan matching as known as the normal distribution transform (NDT) [5] as our scan matching method due to its advantages reported in [6], [7] that robustness of using NDT scan matching is higher than the use of point-to-point iterative closest point (ICP) [8] and point-to-plane ICP. To summarize, we proposed to test 3D NDT scan matching using the INS/GNSS/3D NDT-SLAM integration scheme in GNSS challenging environment.





#### II. PROPOSED METHODOLOGY

To improve performance of pose estimation and to mitigate the error from conventional INS/GNSS integration scheme in such GNSS challenging environment. We perform a multi-sensor fusion using LiDAR-based NDT scan matching with INS/GNSS integration scheme. Fig. 1 shows a workflow of our proposed integration scheme.

## A. INS/GNSS/NDT-SLAM Integration Scheme

As shown in Fig. 1, we proposed the INS/GNSS loosely coupled integration scheme with our proposed 3D NDT LiDAR-SLAM. The Kalman Filter (KF) consists of two models; a system and measurement model. The system model estimates the behavior of the state over time while the measurement model is used to describe how measurements correspond to the state from the system model. The discretetime form of a INS system model is expressed as (1).

$$x_{k+1} = \Phi_{k,k+1} x_k + w_k \tag{1}$$

Where the subscript k represents the time epoch,  $x_{k+1}$  and  $x_k$  are the state vector at epoch k+1 and k, respectively.

 $x = \begin{bmatrix} \delta r & \delta v & \delta & b_g & b_a & s_g & s_a \end{bmatrix}_{21 \times 1}^T$  is the state vector, including the components of position, velocity, attitude error, biases and scale factors of gyroscope, and accelerometer, respectively.  $\Phi_{k,k+1}$  is the transition matrix and  $w_k$  is the process noise due to the presence of the input white noise during the time interval  $(t_k, t_{k+1})$ .

The discrete-time form of a measurement model is expressed as (2).

$$z_k = H_k x_k + \epsilon_k \tag{2}$$

Where  $H_k$  is the design matrix at epoch k,  $z_k$  is the measurement vector at epoch k,  $\epsilon_k$  is the measurement noise due with zero mean, and  $R_k$  is a variance covariance matrix.

This paper adopts NDT-derived velocity and heading as major measurements. Error measurement equations can be written as (3) and (4).

$$\delta z_v = \hat{v}_{nav}^n - \check{v}_{ndt}^n \tag{3}$$

$$\delta z_{\varphi} = \hat{\varphi}_{nav}^n - \check{\varphi}_{ndt}^n \tag{4}$$

Where v is the forward velocity of test vehicle and  $\varphi$  is the heading.  $\hat{v}_{nav}^{n}$  and  $\check{v}_{ndt}^{n}$  are the estimated forward velocities from INS/GNSS and NDT-based LiDAR-SLAM, respectively. While  $\hat{\varphi}_{nav}^n$  and  $\check{\varphi}_{ndt}^n$  are the estimated heading from INS/GNSS and NDT-based LiDAR-SLAM, respectively.

#### B. NDT Scan Matching

The goal of NDT scan matching is to find the pose of the current scan that maximizes the likelihood that points of the current scan lie on the reference scan. The first step is to subdivide the space occupied by the scan into a grid of cells (voxels in 3D case). A probability density function in each cell can be then interpreted as a generative process for point  $\vec{x}$  within the cell. Therefore, the likelihood of having measured  $\vec{x}$  is written as (5)

$$p(\vec{x}) = \frac{1}{(2\pi)^{D/2}\sqrt{|\Sigma|}} \exp(-\frac{(\vec{x} - \vec{\mu})^T \Sigma^{-1} (\vec{x} - \vec{\mu})}{2})$$
(5)

Where  $\vec{\mu}$  and  $\Sigma$  denote as the mean vector and covariance matrix of the reference scan points within the cell where  $\vec{x}$  lies. The mean and covariance are computed as (6) and (7).

$$\vec{\mu} = \frac{1}{m} \sum_{k=1}^{m} \vec{y}_k \tag{6}$$

$$\Sigma = \frac{1}{m-1} \sum_{k=1}^{m} (\vec{y}_k - \vec{\mu}) (\vec{y}_k - \vec{\mu})^T$$
(7)

Where  $\vec{y}_{k=1,\dots,m}$  are the positions of the reference scan points contained in the cell.

To optimize the transform parameters  $\vec{p}$  (rotation and translation) from pose estimate of the current scan, the best  $\vec{p}$  should be the one that maximize the likelihood function in (8)

$$\Psi = \prod_{k=1}^{n} p(T(\vec{p}, \vec{x}))$$
(8)

Where  $T(\vec{p}, \vec{x})$  denote as transformation function. The NDT score function  $s(\vec{p})$  for the current parameter vector is expressed as (9)

$$s(\vec{p}) = -\sum_{k=1}^{n} \tilde{p}\left(T(\vec{p}, \vec{x})\right)$$
(9)

To find the parameter  $\vec{p}$  that optimize  $s(\vec{p})$ , Newton's algorithm can be used to iteratively solve the equation  $H\Delta \vec{p} = -\vec{g}$ , where H and  $\vec{g}$  are the Hessian matrix and gradient vector of NDT score function. The increment  $\Delta \vec{p}$  is added to the current pose estimate in each iteration, so that  $\vec{p} \leftarrow \Delta \vec{p} + \Delta \vec{p}$ . More details can be found in [9].



Fig. 2. Overview of proposed 3D NDT LiDAR-SLAM

Fig. 2 shows our current development on 3D NDT LiDAR-SLAM algorithm that is based on four main modules which can be briefly introduced as follows:

- Preprocessing: main task is to preprocess an input • scan of point cloud using four sub-processes such as limit the point cloud range, ground removal, de-noise, and down-sample. The input is a raw point cloud, while the output is a preprocessed point cloud.
- Scan matching: main task is to perform NDT scan matching for estimating transform with aiding of initial guess. The input and output is the preprocessed point cloud and the transform information.
- Refinement: main task is to refine the pose by re-• computing transform at 1 Hz, constructing an incremental map and extracting sub-map for next scan matching pair. The inputs are the raw point cloud, preprocessed point cloud, and the transform information, respectively.
- Smoothing: main task is to de-noise an outlier from the NDT-derived measurements. In this paper, we proposed to use the wavelet de-noising method. The input and output is the NDT-derived measurements, and the de-noised measurements, respectively.

## **III. EXPERIMENTS**

# A. Test System

Fig. 3 shows the sensor configuration on the experimental platform we used in this paper. Navigation sensors of test system includes a tactical grade IMU from Novatel IMU FSAS (see details in TABLE I.), laser scanner from Z+F Profiler, and one GNSS antenna from Novatel ProPak6. To test our proposed method, we only use single laser scanner from Velodyne VLP-16 this is mounted on top of this mobile mapping system (MMS).



Fig. 3. Experimental platform

TABLE I. SPECIFICATION OF NOVATEL IMU FSAS

Characteristics	Accelerometer	Gyroscope	
Bias Instability	$\leq 1 \ \mu g$	$\leq 0.75^{\circ}/h$	
Random Walk Noise	-	$\leq 0.1^{\circ}/\sqrt{h}$	

### B. Reference System

To validate our system on how accurate it is compared with the conventional method, we use a high navigation grade IMU from iNAV-RQH (see details in TABLE II.) as shown in Fig. 4 to generate the reference POS or ground truth through the commercial INS/GNSS processing software Inertial Explore (IE) version 8.70.



Fig. 4. Navigation grade iNAV-RQH

TABLE II.	SPECIFICATION OF INAV-RQH
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Characteristics	Accelerometer	Gyroscope		
Bias Instability	< 15 µg	< 0.002°/h		
Random Walk Noise	$8 \ \mu g / \sqrt{Hz}$	$0.0018^{\circ}/\sqrt{h}$		

#### C. Experimental Environment

Two experiments were conducted in Tainan City, Taiwan, on October 6, 2020 by the test system MMS car as shown in Fig. 3. An overview image of the first and second experiment are shown in Fig. 5 and 6 in semi-open sky near and at THSR Shalun Station, respectively. The total travelled distance of the first and second experiment is about 9,410 and 9,872 meters, respectively. While the vehicle speed for both experiments was set at 30 km/hr. The route starts at the first alignment part and then travels (as navigated by yellow arrows) into the test part (blue-colored line) with two rounds, and turns back to the second alignment part at final. To sum up, only trajectory excluded the alignment parts will be used to analyze the performance of our proposed method.



Fig. 5. Bird's eye view of the first experiment



Fig. 6. Bird's eye view of the second experiment

IV. RESULTS AND DISCUSSION

#### A. First Experiment: Near the THSR Shalun Station

As the statistics shown in TABLE III, there is no any improvement can be performed in this experiment from our proposed method. It is worth mentioning that NDT scan matching is significantly sensitive to the initial guess and an observed environment, especially where an observed scene was contained with denser trees rather than man-made structures. NDT cannot help to improve the positioning accuracy for the conventional method.



Fig. 7. 2D Trajectory comparison of the first experiment

Error (m.)	INS/GNSS			INS/GNSS/NDT		
	Е	Ν	U	Е	N	U
Max.	1.25	1.22	3.21	1.25	1.22	3.21
Average	0.01	0.08	0.18	0.01	0.08	0.18
STD	0.29	0.22	0.68	0.29	0.22	0.68
RMSE	0.29	0.23	0.70	0.29	0.23	0.70
2D & 3D	2D 0.37		3D 0.79	2D 0.37		3D 0.79
Improvement	-		-	0%		0%

TABLE III. STATISTICAL ANALYSIS OF POSITIONING ERROR FOR FIRST EXPERIMENT (SECOND ROUND)

#### B. Second Experiment: At the THSR Shalun Station

According to page limitation, we can only show the statistical analysis for the second round part. As the statistics shown in TABLE IV, there are significant improvement can be performed in this experiment from our proposed method. Positional RMSE reach up to only about 0.24 meters (79%) and 0.38 meters (69%) in 2D and 3D, respectively. It is worth mentioning that major key for this improvement should come from an observed environment. Because of this experiment rich more uniform and continuous feature points with man-made structures of THSR station compared with the first experiment. In NDT scan matching, we trust believe that these good geometric point clouds can help to compensate the effect of poor initial guess, improve performance of the pose estimate and mitigate the drifts from conventional method, eventually.



 
 TABLE IV.
 Statistical analysis of positioning error for Second experiment (second round)

Error (m.)	INS/GNSS			INS/GNSS/NDT		
	Е	N	U	Е	N	U
Max.	3.76	0.79	1.81	0.58	0.50	0.89
Average	0.42	0.04	0.06	0.07	-0.04	-0.05
STD	1.00	0.24	0.55	0.16	0.15	0.29
RMSE	1.09	0.25	0.55	0.18	0.16	0.30
2D & 3D	2D 1.12		3D 1.25	2D 0.24		3D 0.38
Improvement	-		-	79%		69%

### V. CONCLUSIONS AND FURURE WORKS

This study has been tested a performance analysis of INS/GNSS integration scheme with NDT scan matching in such a GNSS challenging environment. The testing results show that initial guess and observed environment play as major keys for LiDAR-based NDT scan matching. In particular, in GNSS challenging environment that given initial guess is relatively poor. Using denser point clouds and observing in rich environment with more uniform and continuous features point clouds from man-made structures can compensate and mitigate the misalignment from the poor initial guess effect. For future work, the frameworks can be further extended to add more information, such as find an appropriate number of scan frame for building submap as reported by [10], avoid the failing of NDT scan matching in some highly dynamic movement or in highly dense traffic as reported by [11], consider to use more laser scanners attached with different direction, and more various scenarios should be addressed and investigated accordingly.

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